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RADIATION PROJECT PROGRESS REPORT NUMBER 5

PARAPOTENTIAL FLOW

by

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PARAPOTENTIAL FLOW

It has long been recognized that the description of equilibrium electron flow in the customary terms in which the self-magnetic field of the beam is neglected is only a poor approximation in the case of intense relativistic beams. The notion of parapotential flow, or flow along equipotential lines, is put forward as the other limiting case for the purpose of understanding and designing diodes and diode regions for very energetic beams of high current. This has been gone into only in the simplest conditions, in which all accelerations are neglected and in which there is axial symmetry. The neglect of accelerations implies that the motion of an individual electron is confined to a specific R-z plane. It can then be shown that the potential is governed by the equation

$$\nabla^2 V = 2R^{-2}\, dI^2/dV, \tag{1}$$

where R is the polar radius, V is the potential, and I is the current contained within the equipotential surface defined by V. The unknown function dI^2/dV is partially determined by the necessity for introducing the parapotential condition $E = \beta H$ after one integration of this equation.

Two cases have been examined, one in which the z-variation is negligible, and the other in which the R-variation is negligible. In the former case we write

$$R^{-1}\partial/\partial R(R\partial\gamma/\partial R) = R^{-2}d\mu^2/d\gamma,$$

where γ is the total electron energy in mc^2 units and μ is the current in units of 17 kA. After one integration we have

$$(\partial\gamma/\partial R)^2 = 4R^{-2}(\mu^2 - \mu_o{}^2),$$

where μ_o is in general an arbitrary function of z. In the present notation the condition $E = \beta H$ becomes

$$\partial\gamma/\partial R = 2\beta\mu/R,$$

so that $\beta^2 = 1-\gamma^{-2} = 1-\mu_o{}^2/\mu^2$, or

$$\gamma = \mu/\mu_o, \tag{2}$$

and $d\mu^2/d\gamma = 2\mu_o{}^2\gamma$; thus the original equation is linear. The complete solution is

$$\gamma = \cosh[2\mu_o \log (R/r_o)], \tag{3}$$

which involves two arbitrary functions of z, r_o and μ_o. On $R = r_o$ the electron energy and the electric field are zero, and therefore the interpretation of μ_o is that it is a current which must flow <u>within</u> the cathode surface to produce the required conditions of the flow. If μ_o is larger than that value which just satisfies (3) with $\gamma = \gamma_a$ (the applied anode potential) and $R = r_a$ (the anode radius), one requires

$$\hat{\gamma} = \cosh (2\mu_o \log \hat{R}/r_o),$$

$$(\gamma_a - \hat{\gamma})/(\hat{R} \log r_a/\hat{R}) = (2\mu_o/\hat{R}) \sinh (2\mu_o \log \hat{R}/r_o)$$

in order to satisfy the potential and field continuity conditions at the beam surface, where conditions are denoted by the circumflex. If μ_o is less than this critical value, the flow is impossible. If μ_o is equal to the critical current μ_c, given by

$$\mu_c = (\text{arc cosh } \gamma_a)/2 \log (r_a/r_o), \tag{4}$$

the volt-ampere characteristic (conductance in mhos) is

$$i/V_a = (1/15)(1 - 1/\gamma_a)^{-1} \text{ arc cosh } \gamma_a/\log (r_a/r_o). \tag{5}$$

This or higher conductances may be regarded as representing parapotential flows. The internal current is included in this value since it is generally supplied by the same source as the sheath current and results in electrons of the same energy (see Fig. 1). In nonrelativistic flows the internal, or core, current makes up most of the flow; the opposite holds for highly relativistic flows. In the particular case $\mu_o = \mu_c$ the minimum conductance occurs for $\gamma_a \sim 2.5$, and the corresponding conductance is

$$0.174/\log (r_a/r_o) \text{ mhos}. \tag{6}$$

This minimum is very broad, and little error is incurred over the range 250 keV $< V_a <$ 2 MeV if this figure is used.

In the alternative case where the R-variation can be neglected it is readily shown that the complete solution is

$$\gamma = \cosh[2\mu_o(X - z/R)], \tag{7}$$

which involves the two arbitrary functions of R, X and μ_o, which in turn depend on the cathode and beam surface shapes. If the cathode is at $z_o(R)$ and the beam surface is at $\hat{z}(R)$, then z_o = XR and

$$\hat{z} - z_o = (R/\mu_o) \text{ arc cosh } \hat{\gamma}.$$

In the case of a metallic cathode μ_o is constant, so that a constant $\hat{\gamma}$ implies $\hat{z} - z_o \propto R$. This is an obvious limiting case of the coaxial flow just discussed, in which the separation of the equipotentials is simply proportional to R. The ratio d_o/r_o (Fig. 2) continues to play the role of an impedance, which can from previous considerations be taken to be the order of

$$5.74\ d_o/r_o \text{ ohms.} \tag{8}$$

This is the highest impedance giving true parapotential flow for this geometry. This concept furnishes a convenient way of matching the diode to a given source impedance. The arbitrary taper is likewise useful in concentrating the flow and confining the angular spread of the beam.

The case of the parallel-plane vacuum pinch diode cannot be treated in terms of these simple models since there is no conical metallic cathode and the coincidence of flow lines and equipotentials can therefore not be satisfied throughout the flow. It is presently conjectured, however, that the flow in this tube is divided into an orthopotential (flow predominantly across equipotentials) core and a parapotential sheath, the configuration of the latter being such as to satisfy approximately the conditions derived for the conical metallic cathode down to some suitable potential, say γ_1. Current is then injected into the parapotential portion from an outside ring (Fig. 3). The potential γ_1 is taken considerably below γ_a, so that it gives rise to a roughly conical core. This core provides the required $\mu_o(R)$ to satisfy the $E = \beta H$ condition in the sheath. The regulatory mechanism for this consists in the self-adjustment of the potential γ_1 so as to provide suitable Child's-law conditions at the cathode, and we should therefore expect that the overall conductance is given in order of magnitude by the naive expression which equates the interelectrode separation with the maximum penetration of an electron in the approximately constant electric field V_a/d and magnetic field $2i/r_o$; the resulting conductance in mhos can easily be shown to be

$$0.017(1 + 1/V_{MV})^{\frac{1}{2}}(r_o/d), \qquad (9)$$

where V_{MV} is V_a expressed in megavolts. Because of the orthopotential character of the core flow it is expected that core electrons will cross sheath trajectories, complicating the description of the flow. One would in fact expect that in attempts to penetrate the sheath the core electrons will encounter very large magnetic fields and thus be reflected back into the core (snowplow effect). It is expected that such effects should become less important with increasing energy, where less core current is required to satisfy the parapotential conditions at the sheath.

At least two computer programs[1,2] have been developed for self-consistently solving such diode problems. These, however, have been plagued by instability in attempting to treat the vacuum-pinch diode. We are working on various mathematical techniques by which these instabilities can be overcome so that good perveance, cathode loading, and trajectory data can be obtained for these devices.

Three parapotential diodes have been designed and two have been built. The two that have been constructed are for the 40-Ω and 7-Ω, 50 ns pulse generators described in references 3 and 4. These are shown in figures 4, 5, and 6. The cathodes were sent to Mr. W. Crewson of EG&G, who kindly had them tungsten-sprayed for us by AVCO.

The 40-Ω diode (P-1) is actually not a good match to a 40-Ω system according to (8), and the flow in this tube is in fact probably not in accordance with these idealistic models; it would, indeed, be hard to design a truly 40-Ω diode with the available geometry. Preliminary results on this tube, shown in Figs. 7, 8, 9, and 10, have been encouraging,however. The 7-Ω diode (P-2) is not a constant-impedance structure; it should be within a factor of 2 of matching according to (6) over about the last half of its length. The coaxial diode (P-3) shown in Fig. 11 is intended primarily for diagnostics. It is provided with a variety of probes for examining the current distribution. Its impedance according to (6) is 5.6 Ω, representing a compromise between the design impedance of 7 Ω and various mechanical considerations. It should be available for testing about 8/1/68.

FIGURE CAPTIONS

1. Schematic of coaxial parapotential diode.
2. Conical parapotential diodes of about equal impedance.
3. Schematic representation of vacuum-pinch flow.
4. Layout of conical diode (P-1) used with the 40-ohm generator.
5. Layout of conical diode (P-2) used with the 7-ohm water Blumlein.
6. Diode of figure 5 assembled to 7-ohm generator.
7. Anode damage in tantalum with 40-ohm generator. Estimated electron energy density ~100 cal/cm^2.
8. Anode damage in titanium.
9. Voltage and current with P-1 cathode. Voltage = 240 kV, current = 10 kA, pulse length = 50 nsec. Impedance change during pulse is substantially less than with needle array cathode.
10. Material buildup on cathode with aluminum anode.
11. Layout of coaxial diode (P-3).

REFERENCES

1. William B. Herrmannsfeldt, "Poisson Equation Solving Program," Stanford Linear Accelerator Center Report Number 51, September, 1965.

2. J. E. Boers, IEEE Transactions on Electron Devices, July 1965, p. 425, "Digital Computer Analysis of Axially Symmetric Electron Guns".

3. John J. Condon, "Radiation Project Progress Report No. 6,"'Measurement of Bremsstrahlung Radiation Produced with High-Current Diodes and Coaxial Blumlein Generators'," to be published.

4. John D. Shipman, Jr., "Radiation Project Progress"- to be published.

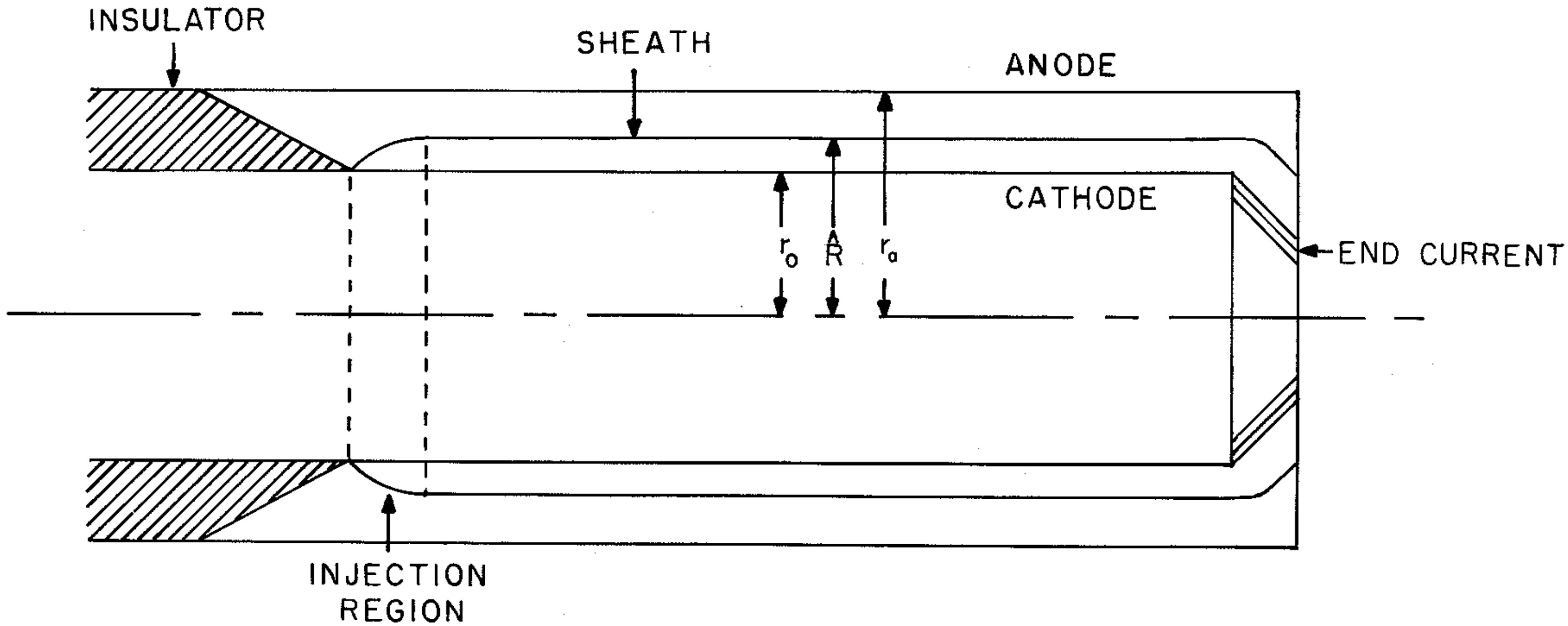

INSULATOR
SHEATH
ANODE
CATHODE
r_o
$\hat{R}$
r_a
END CURRENT
INJECTION REGION

Figure - 1

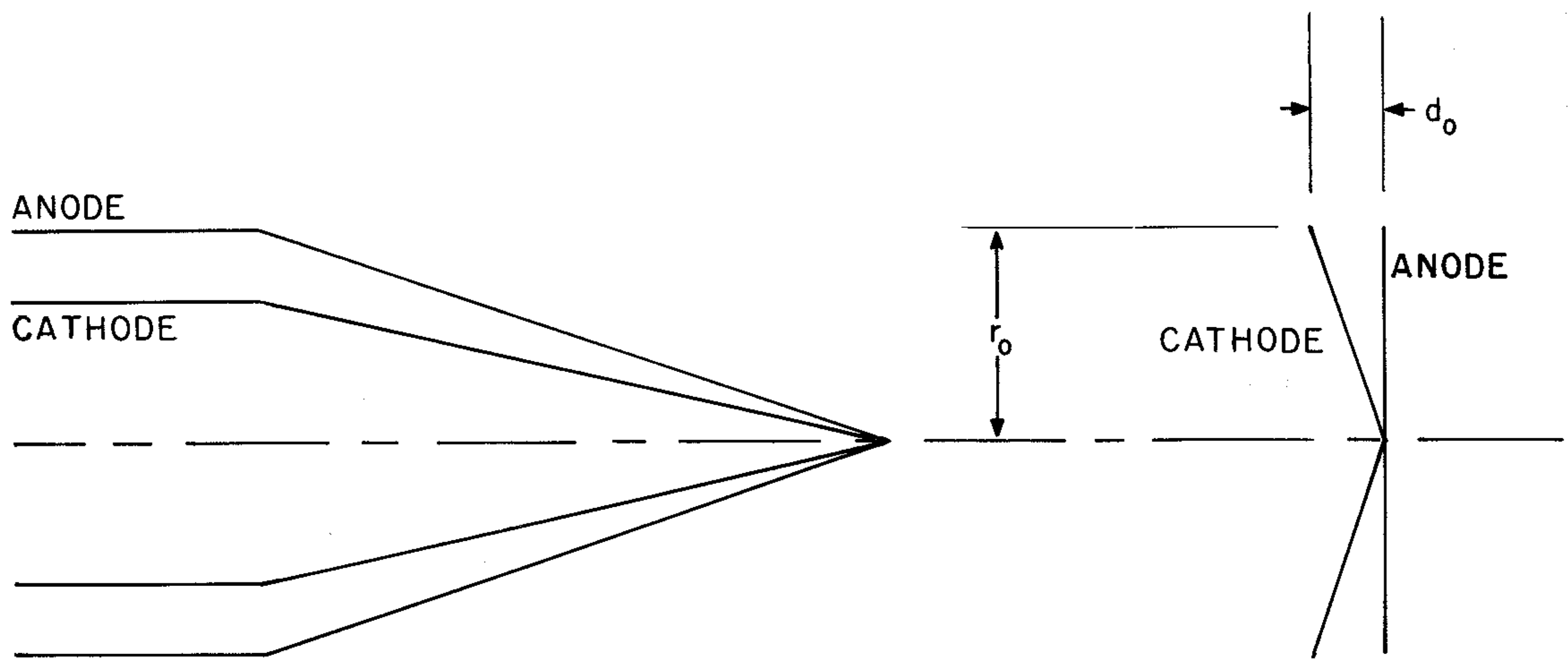


Figure - 2

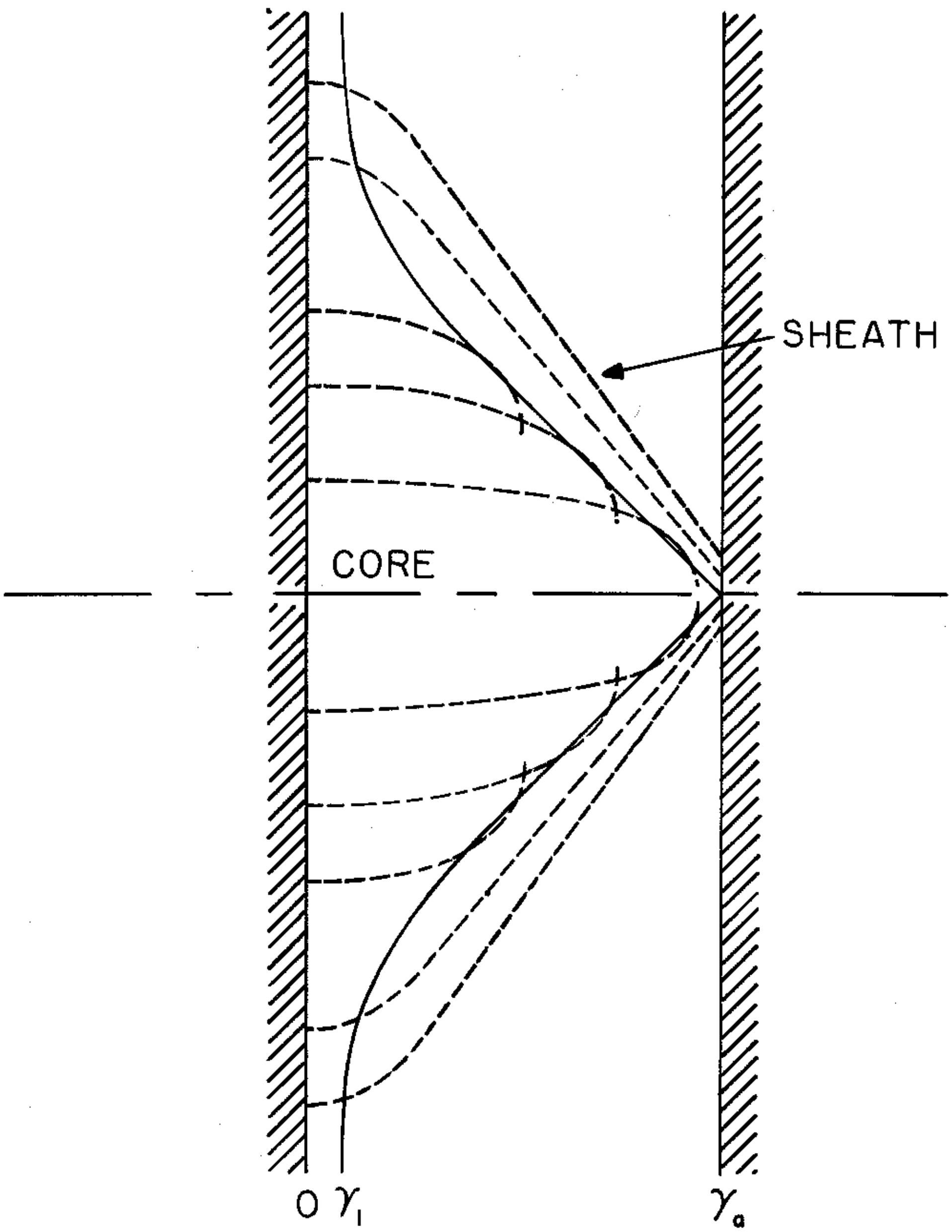
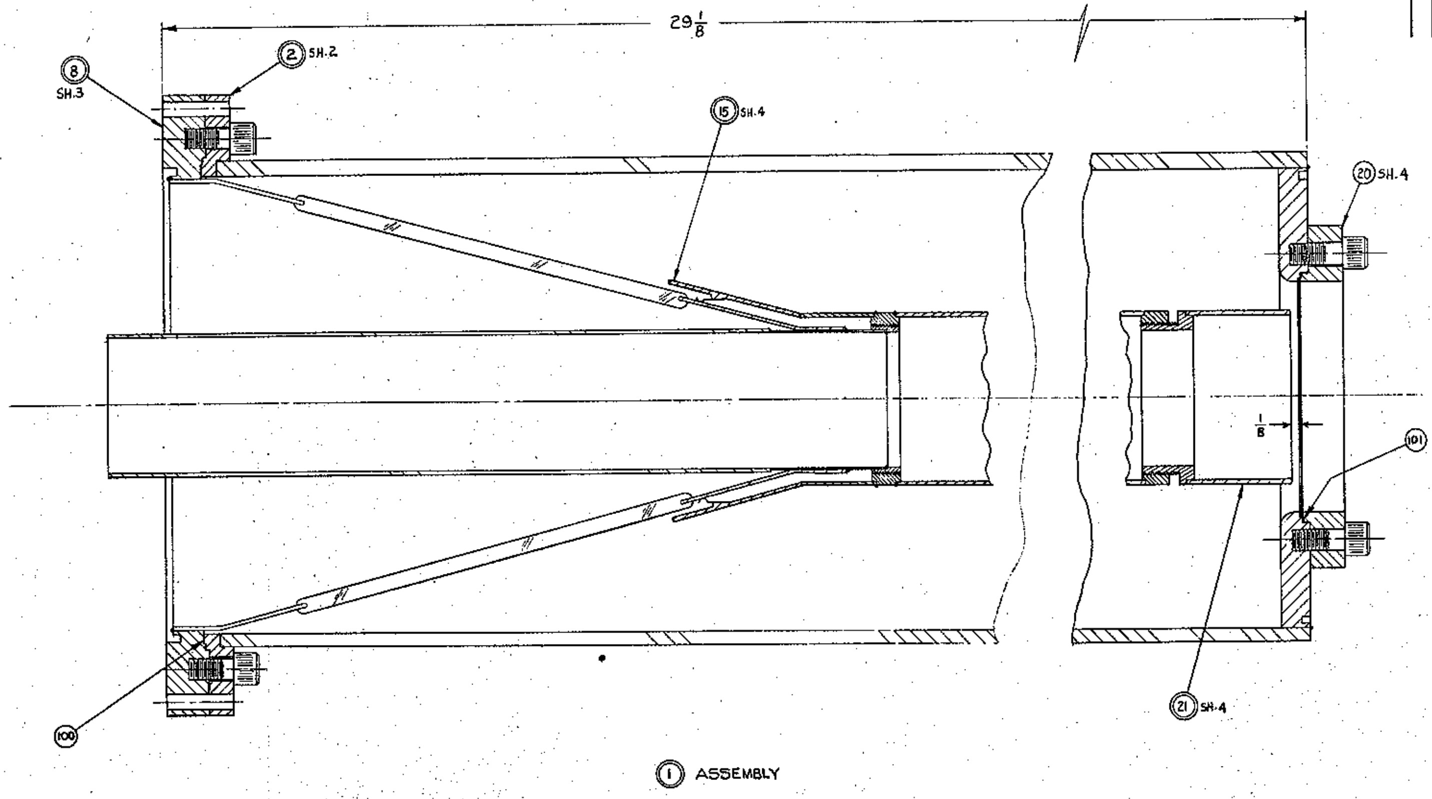


Figure - 3

REVISIONS					
LTR	PT. NO.	BY	DESCRIPTION	DATE	APPROVED



1 ASSEMBLY

PART NO.	DESCRIPTION	SPECIFICATION	QTY REQD
101	COPPER GASKET 4.370 O.D. X 4.00 I.D. X .040 OF HC COPPER		1
100	COPPER GASKET 8.740 O.D. X 8.125 I.D. X .040 OF HC COPPER		1

PARTS LIST

UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES
TOLERANCES ON
FRACTIONS DECIMALS ANGLES
±1/64 ±.005 ±2°
DO NOT SCALE DRAWING

ENGINEER O'CONNELL DATE
BLDG. 101 RM. 167 TEL. 2206
DRAWN L.P. DATE 4/13/69 CHECKED DATE
DESIGN SECTION DATE
E.D. & D. BRANCH DATE
APPROVED BY DATE
APPROVED BY DATE

NAVAL RESEARCH LABORATORY
ENGINEERING SERVICES DIVISION
WASHINGTON D.C. 20390

COAXIAL FLOW GUN (P-3)

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-463
SCALE 1/1 | SHEET 1 OF

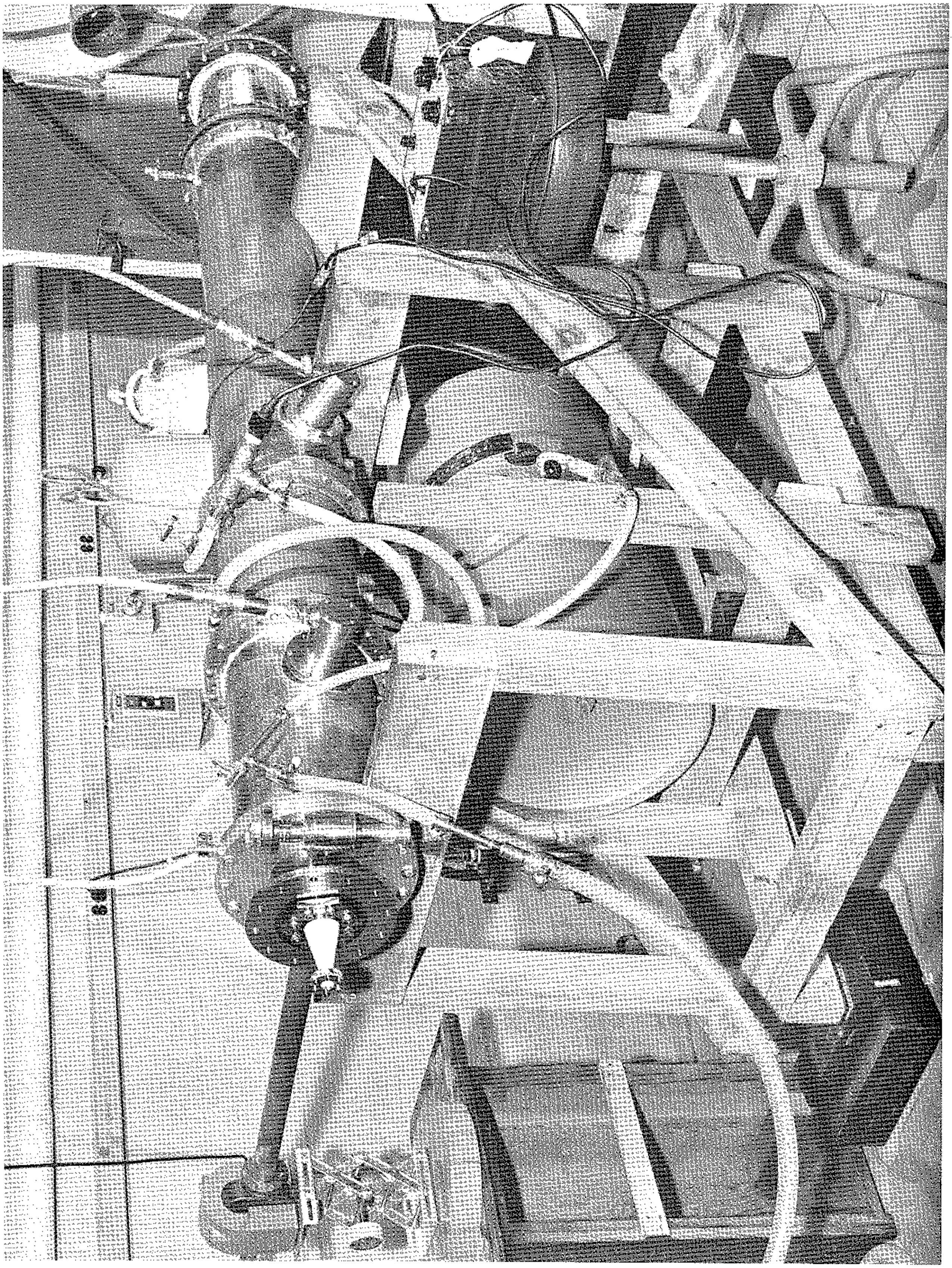
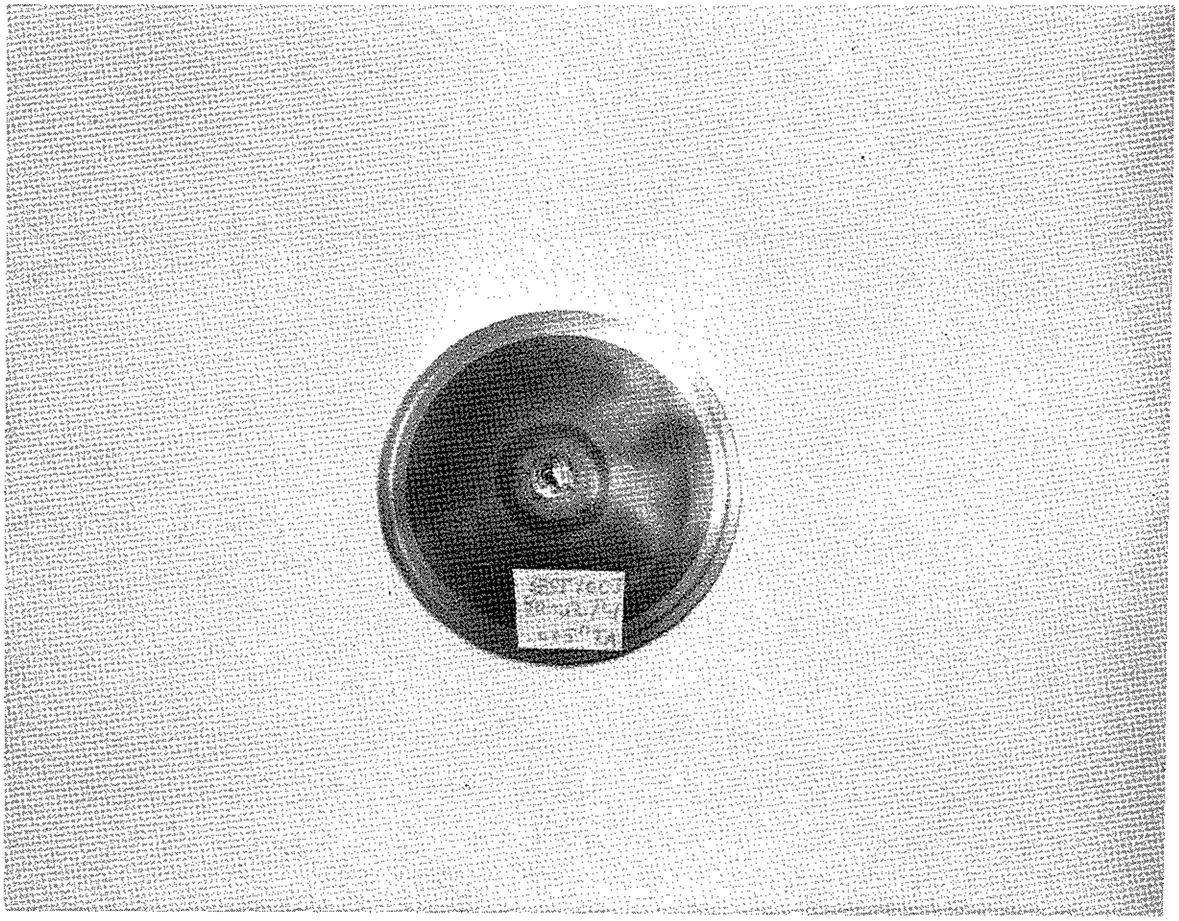


Figure - 2



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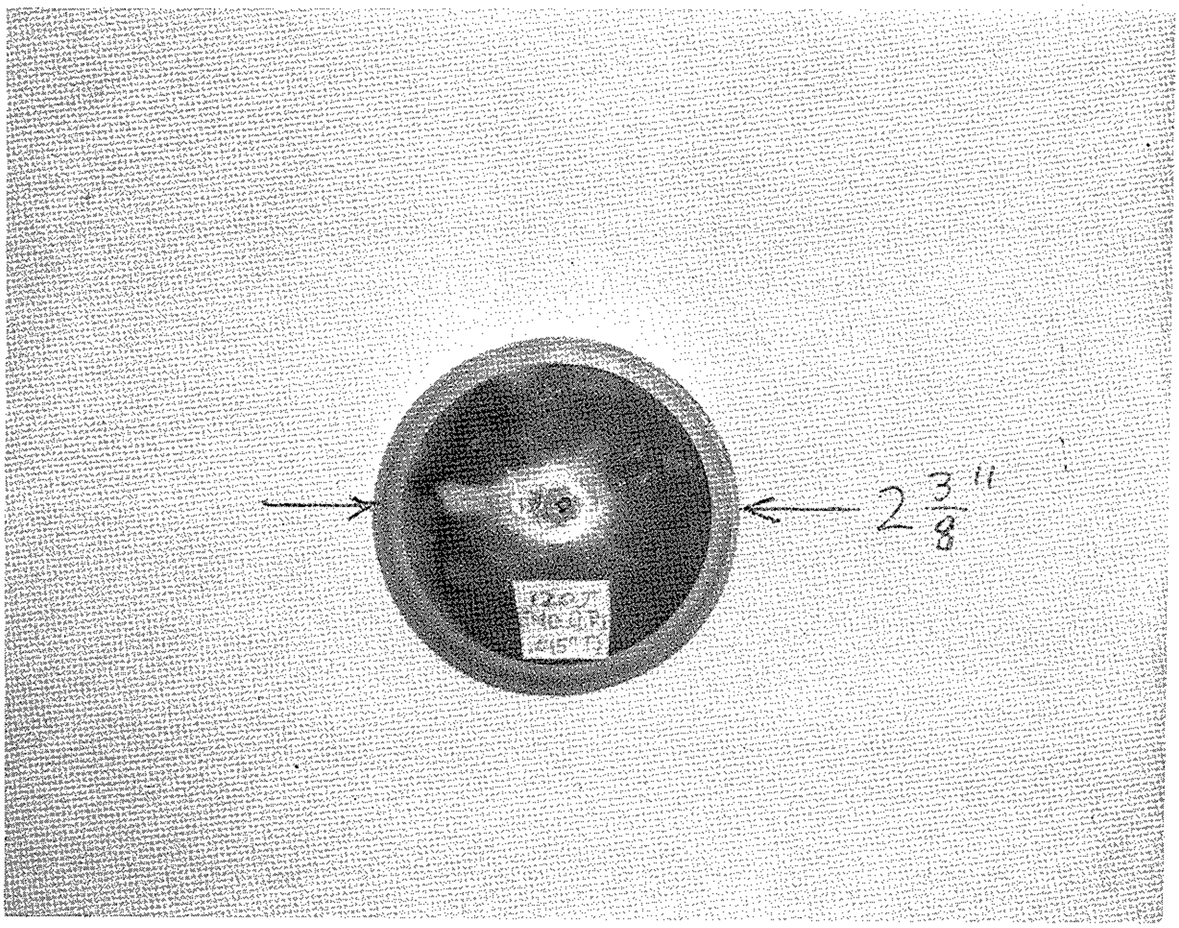


Figure - [illegible]

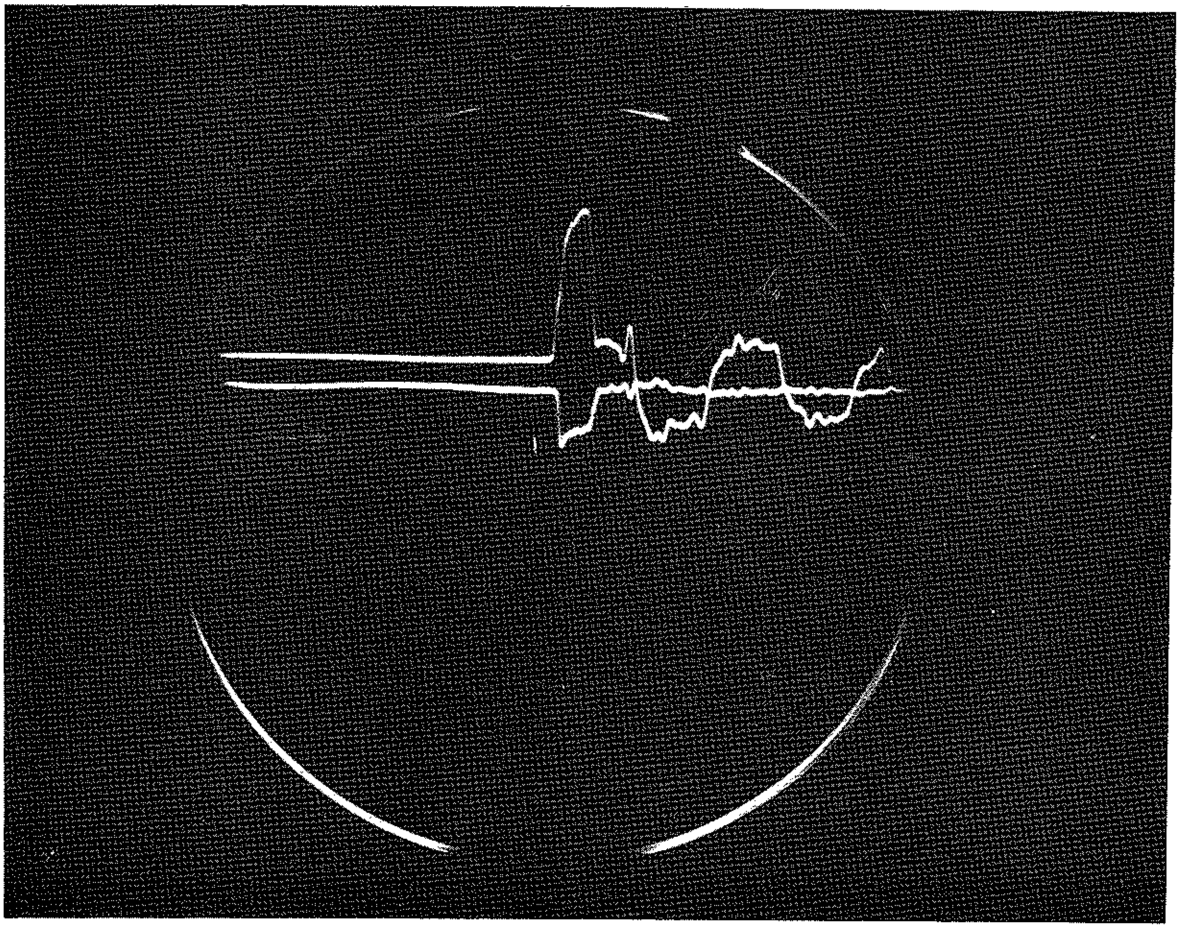
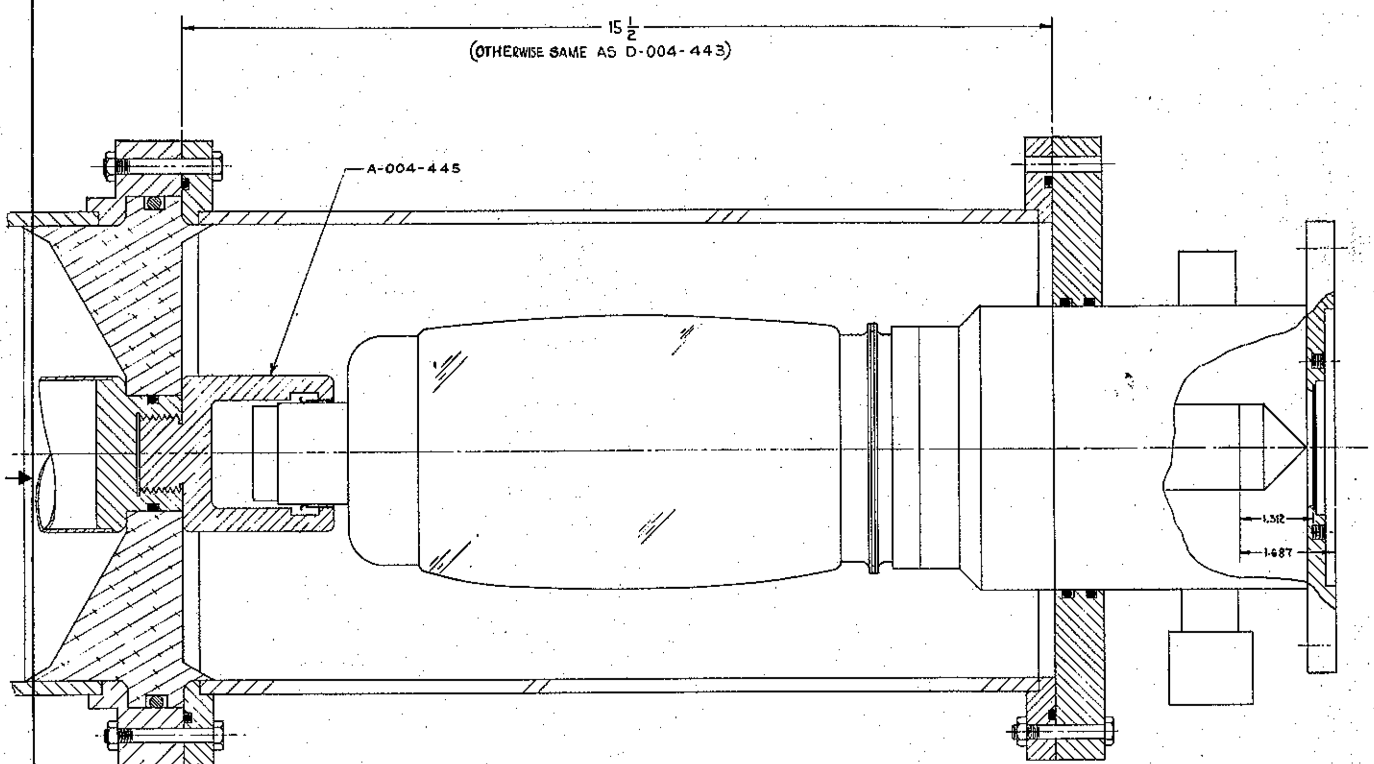


Figure - 9



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40 OHM LINE
TEST SECTION
GUN ASSEM. (P-1 CATHODE)

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-444
SCALE 1/1 | SHEET OF

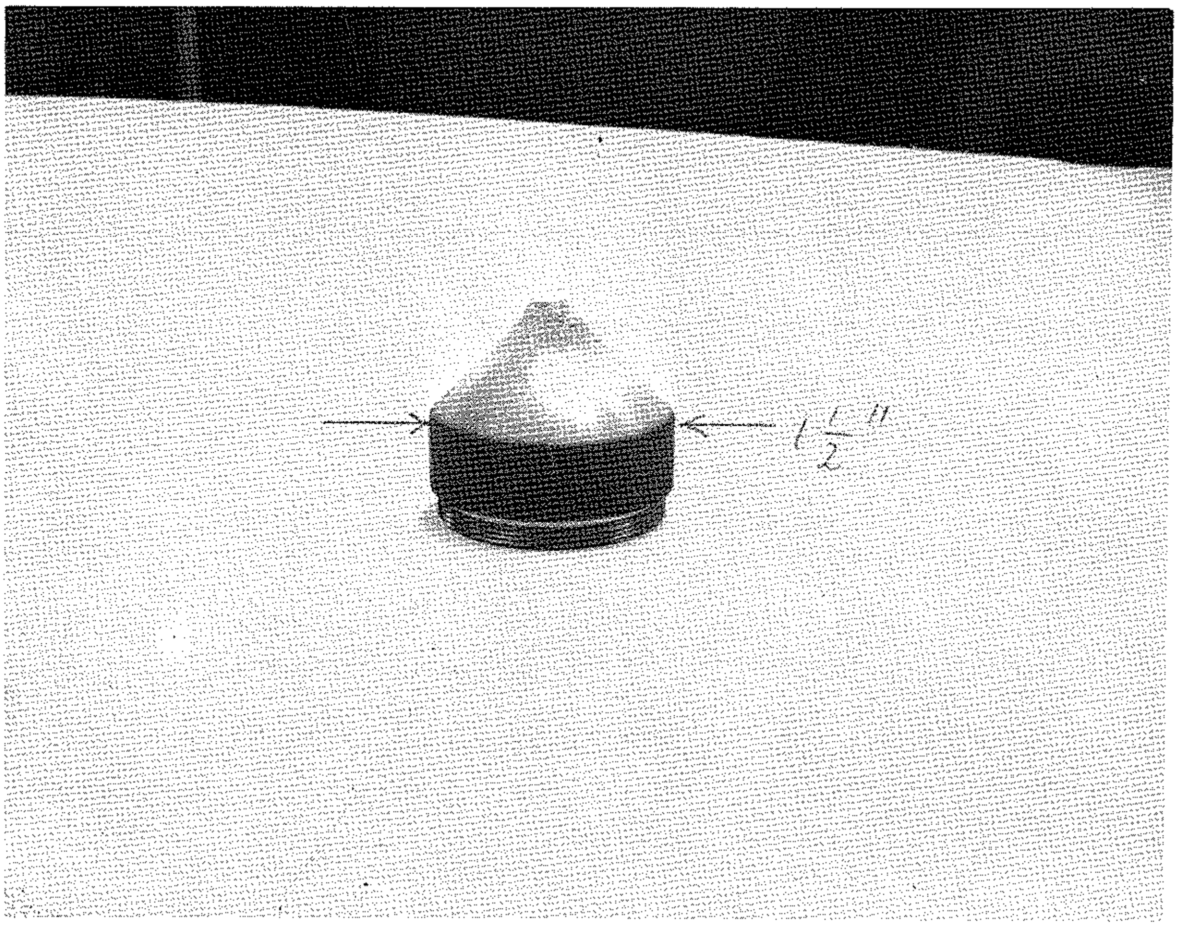

Figure - 12

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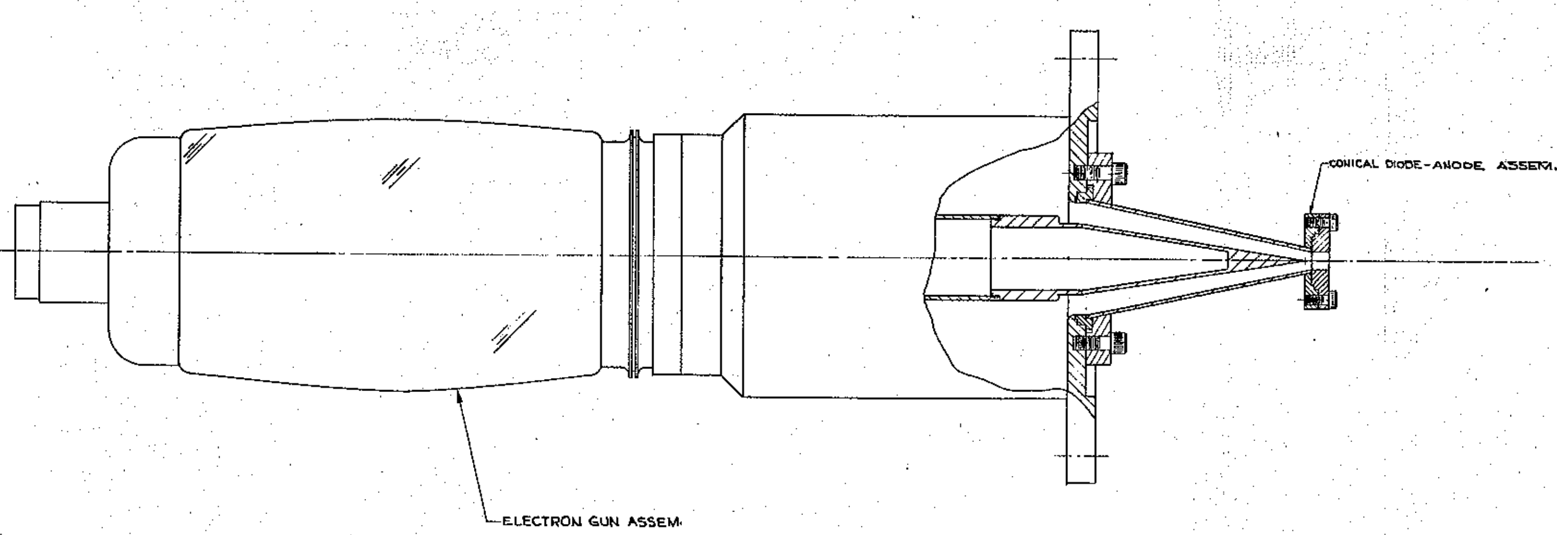

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PARTS LIST

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E.D. & D. BRANCH DATE
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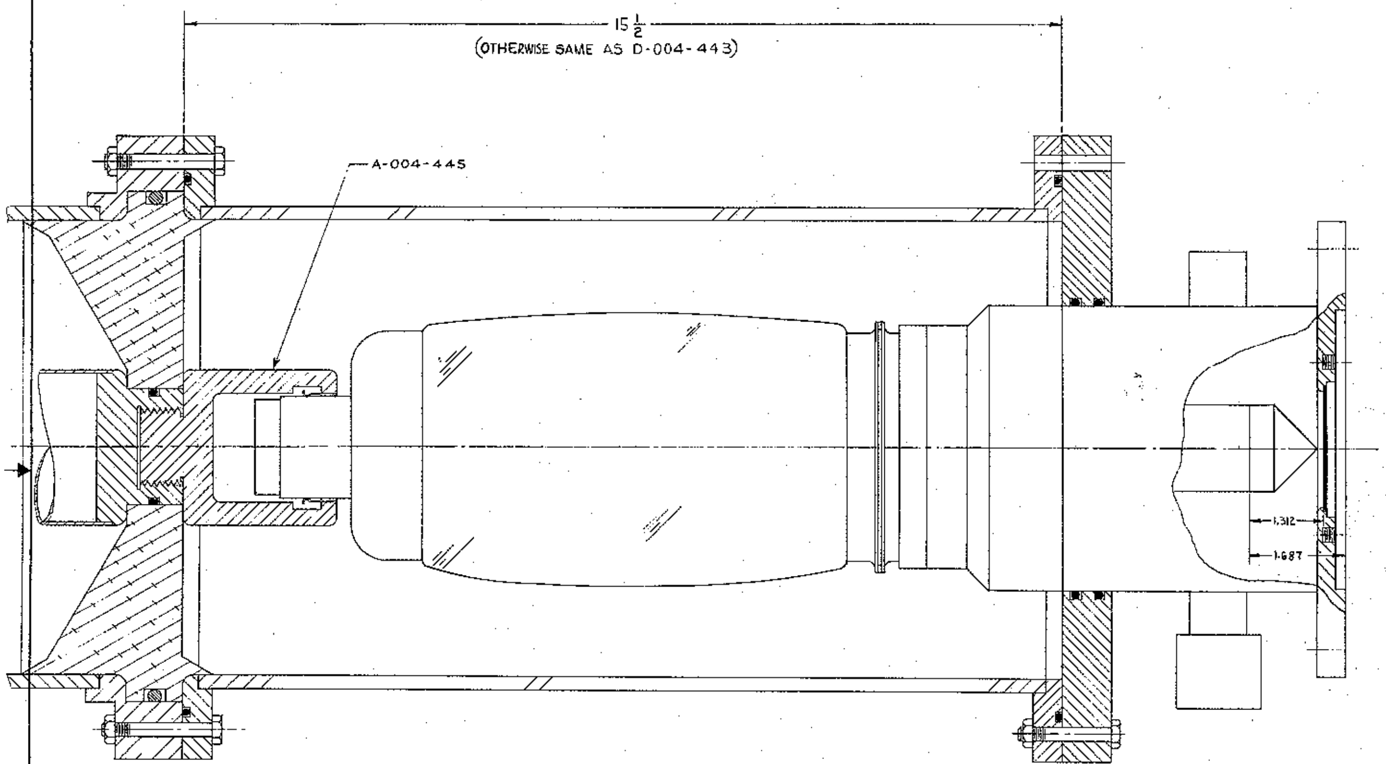
CONICAL DIODE (P-2)
FOR
7 OHM LINE

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-465
SCALE 1/1 | SHEET OF



SUB-ASSEMBLY

LTR	PT. NO.	BY	REVISIONS DESCRIPTION	DATE	APPROVED

PART NO.	DESCRIPTION	SPECIFICATION	QTY REQD

PARTS LIST

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±1/64 ±.005 ±2°
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DESIGN SECTION DATE
E.O. & D. BRANCH DATE
APPROVED BY DATE
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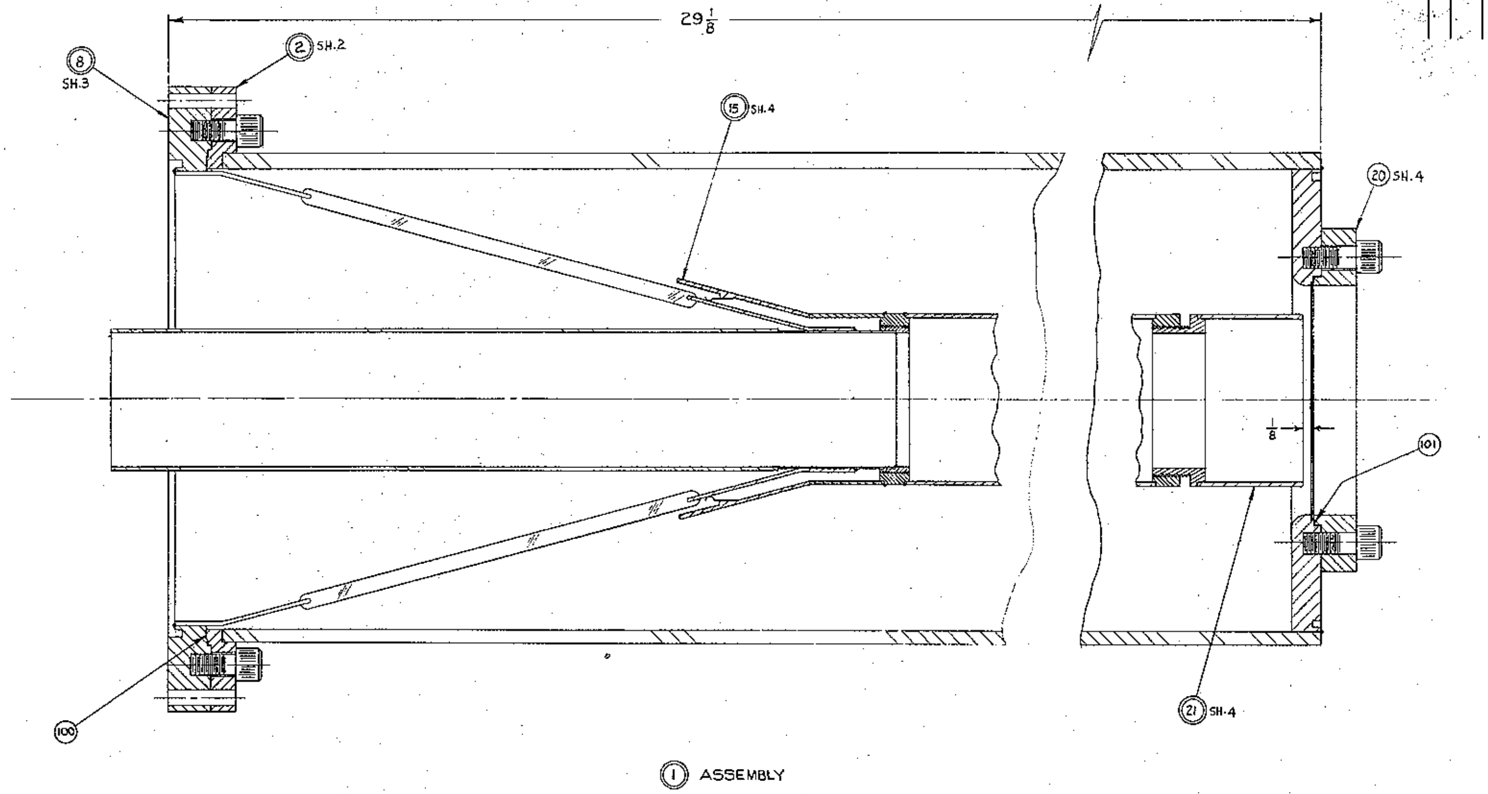
40 OHM LINE
TEST SECTION
GUN ASSEM. (P-1 CATHODE)

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-444
SCALE 1/1 | SHEET OF



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COAXIAL FLOW GUN (P-3)

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-463

SCALE 1/1 | SHEET 1 OF

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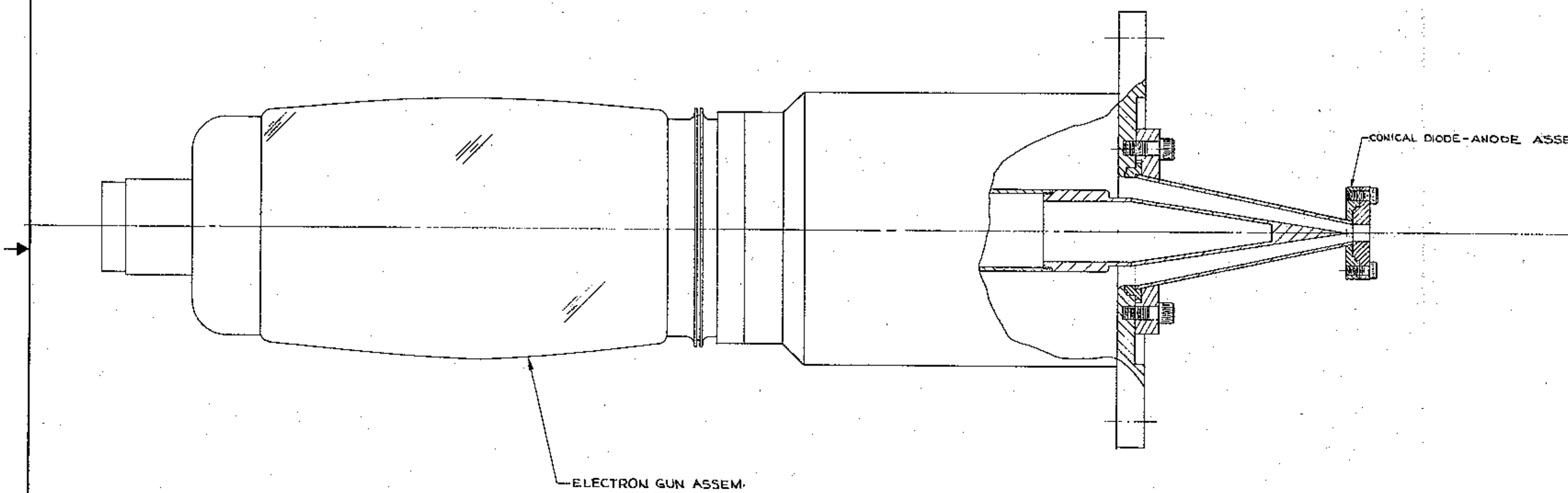

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CONICAL DIODE (P-2)
FOR
7. OHM LINE.

SIZE D | CODE IDENT NO. 81995 | DWG. NO. D-004-465
SCALE 1/1 | SHEET OF